STUDIES ON FATIGUE BEHAVIOR OF STRUCTURAL ELEMENTS FOR AUTOMOBILE*

(Fundamental Study on Statistical Fatigue Strength Characteristics of Friction-Welded Butt Joint Composed of Similar Matallic Material)

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1. INTRODUCTION

Recently the application of non-metallic and inter-metallic composite materials has come into wider use considerably for structural elements for automobile[1) 2)] and other mechanical structures.

In this study reported here, the statistical treatment of the fatigue test results done by using the friction-welded butt joint composed of the similar metallic material, considered as one of the inter-metallic composite materials, is described in detail[3)].

Further increase in demand for friction-welded butt joint is anticipated because of its utility for some engineering purposes and the technical easiness of this welding method to make up the butt joint axes composed of dissimilar metallic materials. However there exist only a few papers concerned with the strength characteristics of this type of joint[4)-13)].

Especially as for the fatigue strength characteristics, there does not exist the precise and the systematical knowledge to establish the reasonable design standard for lacking of the studies on the fatigue problem done with attention to the pecular condition to the friction-welded butt joint[14) 15)]. Such a state of the investigation on this subject seems to be supported with the following reason; That is, it has been recognized emprically that there does not exist any noticable problem on this type of joint especially from the view point of fatigue strenght because the fatigue strength of the friction-welded interface does exceed that of the base material. But, in the fundamental study conducted by the authors using the friction-welded butt joint specimen composed of 0.23%C carbon steel, it was revealed that the fatigue lives of this type of the specimen showed a considerably wide range scattering in comparison with those of the base material specimen[16)].

To clarify the statistical strength characteristics of the friction-welded butt joint specimen in fatigue, a series of the experiments were planned and conducted, where the base material specimen of 0.46%C carbon steel (JIS. S45C) and the friction-welded butt joint specimen were used and experiments were done at the several stress levels in the overstress range.

Statistical discussions on the results obtained were made by accepting the Weibull distri-

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bution as the original distribution of the fatigue life, and it was revealed that there existed distinct difference in the *S-N* relations with the distribution of the fatigue life between both of the specimens used in this investigation, *i.e.*, one was the base material specimen and another the friction-welded butt joint specimen.

2. EXPERIMENTAL MATERIAL AND SPECIMENS

0.46%C carbon steel (JIS. S45C) was used as a base material in this study. The chemical composition, and the condition for the heat treatment and mechanical properties in static tension test are listed in Tables 1 and 2 respectively. This material was supplied as the hot rolled

Table 1 Chemical composition of the material.

Material	Chemical composition (%)							
	C	Si	Mn	P	S	Ni	Cr	Cu
0.46%C carbon steel	0.46	0.23	0.79	0.018	0.018	0.01	0.12	0.12

Table 2 Condition of heat treatment and mechanical properties of the material.

Heat treatment	Upper yield point σ_{SO} kg/mm²	Lower yield point σ_{SU} kg/mm²	Ultimate tensile strength σ_B kg/mm²	Breaking strength on final area σ_T kg/mm²	Elongation δ %	Reduction of area ϕ %
900°C, 1hr., F. C.	33.5	31.6	62.6	98.2	32.6	44.0

bar of 25mm in diameter, and was machine-finished to the specimen shape indicated in Fig. 1 after the annealing treatment, then thus obtained specimens weresupp lied to the experiment as the base material specimen. On the other hand, to obtain the friction-welded butt joint specimen (hereafter, called "joint specimen" for brevity), the material was machined to the shape and dimensions shown in Fig. 2 after the heat treatment done in the same manner as that for the base material specimen, and a pair of these materials were friction-welded under the conditions listed in Table 3. An example of the whole feature of the longitudinal cross-sectional area is represented in Fig. 3. Then, the friction-welded butt joint was machine-finished to the same shape of the specimen as that for the base material specimen. Here, the friction-welded interface is located at the minimum sectional area, or at the root of the semi-cirular notch of the specimen. These joint specimens were supplied to the experiment without any

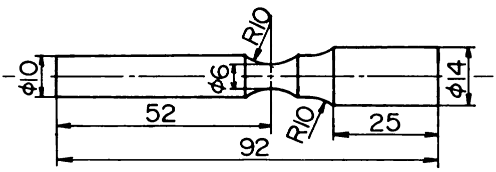

Fig.1 Shape and sizes of the specimen.

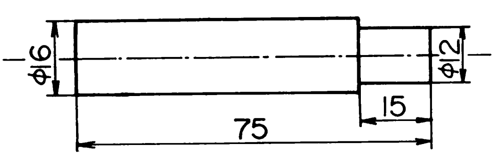

Fig.2 Material for friction welding.

Table 3 Conditions for friction welding.

Heating pressure	P_1 4.0 (kg/mm^2)
Forging pressure	P_2 8.0 (kg/mm^2)
Heating upset	δ_1 5.0 (mm)
Total upset	δ_2 8.2 (mm)
Heating time	T_1 6.0 (sec.)
Forging time	T_2 1.8 (sec.)
Rotating rate	N 2883 (rpm)

heat treatment after machining to the specimen.

Besides, theoretical stress concentration factor due to the existence of the semi-circular notch under the bending load condition is about 1.08.

For the purpose of reference, illustration of the friction welded process is represented in Fig. 4.

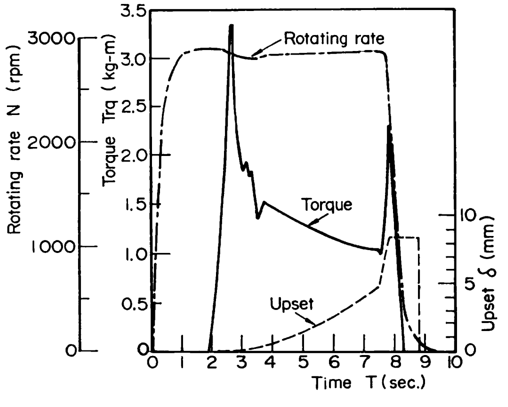

Fig. 4 Schematic representation of friction welding process.

Hardness distribution on the longitudinal cross-sectional area was measured for the friction-welded butt joint with the aid of a micro-Vicker's hardness testing machine, for the material near the friction-welded interface was expected to be hardened because of the segregation of the crystal grains caused by "dynamic recrystalization" which seemed to be occurred during the rapid thermal rise and the strong working in upsetting process. The hardness distridution curves obtained are shown in Fig. 5, where the hardness distributions were measrued

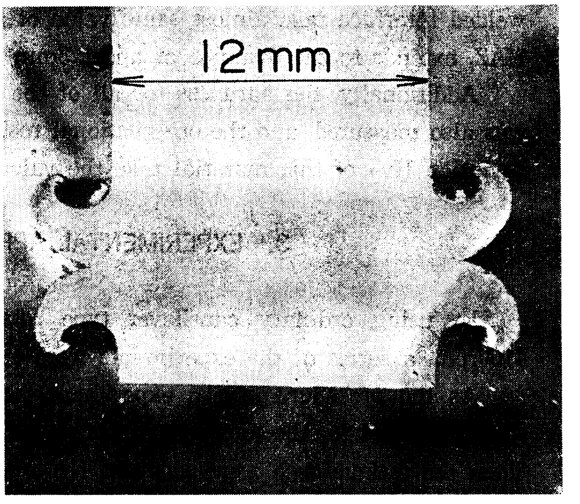

Fig. 3 Whole feature near the friction welded interface.

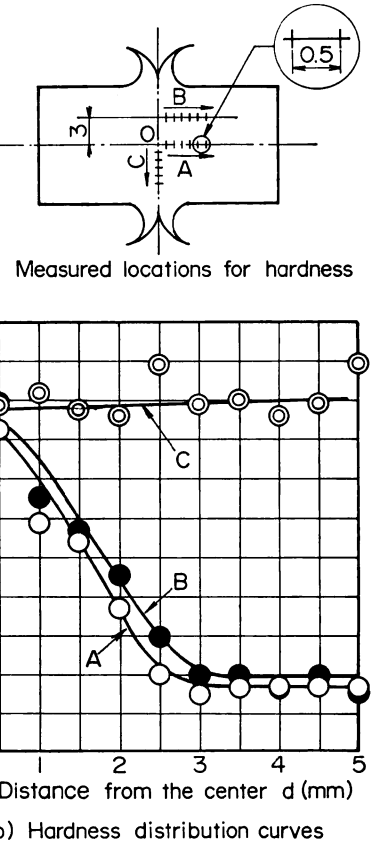

Fig. 5 Hardness distribution near the friction welded interface.

along three different directions. This figure shows that the hardness values along the friction-welded interface take almost same value of about Hv=320, and that the heat affected zone, HAZ, extends to the distance of about 3mm from the friction-welded interface.

Additionally, the hardness for all of the base material specimen used in this experiment was also measured, and the observational reslts show that the mean value $Hv\mu$ and the standard deviation $Hv\sigma$ of this material take the values of about $Hv\mu=188$ and $Hv\sigma=12$ respectively.

3. EXPERIMENTAL APPARATUS AND PROCEDURE

By using ordinary cantilever type rotating bending fatigue testing machine of about 4100rpm, a series of the experiments were carried out in the range of about 10^7 stress cycles.

As the procedure of the experiment, at first, rough *S-N* curves for the base material specimens and for the joint specimens were obtained by using small number of the specimens. Then the fatigue data were obtained by consuming some amount of the specimens at each stress level selected at the interval of 2kg/mm^2 above the fatigue limits of both typtes of the specimens to discuss the fatigue life distribution characteristics of the joint specimen in comparison with that of the base material specimen from the standpoint of Weibull statistics concept.

4. EXPERIMENTAL RESULTS AND DISCUSSIONS

Experimental results obtained in this study are tabulated in Tables 4 and 5 for the base material specimen and for the joint specimen respectively, and are plotted on an *S-N* diagram in Fig.6. In this figure, *P-S-N* curves for $P=20$, 50 and 80% are drawn for both types of

Table 4 Results for the base material specimen.
Number of stress cycles ($\times 10^5$)

No.	Stress (kg/mm^2)				
	32.0	30.0	28.0	27.0	26.0
1	1.457	3.354	5.240	10.045	9.268
2	1.461	3.791	9.374	30.847	30.091
3	1.664	5.831	9.720	48.309	101.991
4	2.190	7.242	11.575	84.588	>114.4
5	2.342	9.500	14.162	>302.3	>176.9
6	2.845	9.653	14.169		>300.5
7	2.875	10.190	19.147		>348.5
8	2.971	10.971	24.457		
9	3.000	14.079	27.460		
10	3.201	20.361	30.956		
11	3.317	20.491	36.716		
12	3.548	23.058	57.401		
13	4.767	24.125	59.381		
14	4.945	36.577	>117.5		
15			>417.1		

Table 5 Results for the joint specimen.
Number of stress cycles ($\times 10^5$)

No.	Stress (kg/mm²) 42.0	40.0	38.0	36.0	34.0	33.0
1	1.246	1.462	2.056	2.519	4.713	31.596
2	1.319	1.705	2.320	4.380	9.911	36.488
3	2.007	2.693	2.857	6.235	11.193	77.352
4	2.151	3.115	4.184	7.595	16.916	>120.0
5	2.369	4.099	4.488	8.228	34.876	>121.5
6	3.543	4.621	4.524	13.063	>119.8	
7	3.742	4.709	5.144	18.747	>120.1	
8	3.893	5.580	6.052	27.450	>125.0	
9	3.980	7.177	6.733	42.086	>175.7	
10	4.000	8.493	13.755	48.865	>180.0	
11	4.825	11.038	>118.52	>110.50		
12	16.209	>116.11	>420.41	>229.5		

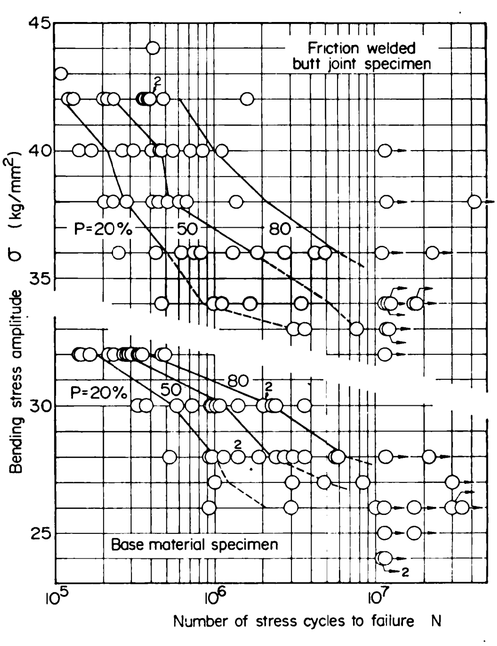

Fig. 6 Experimental results and P-S-N curves.

specimens. Here, the cumulative probability for failure P, the ordinate of Weibull probability paper, was reckoned with the following formula,

$$P=\frac{i}{n+1} \quad \cdots\cdots\cdots\cdots(1),$$

where i is the order of the fatigue life and n the total number of the specimen used at each stress level involving run-out specimens. And fatigue lives corresponding to P=20, 50 and 80% were determined from the curves drawn smoothly to run among the experimental results plotted on Weibull probability paper. This figure shows that the fatigue limits for both types of the specimens are to be estimated to take the values of about 26 and 34kg/mm² (255 and 333MPa) respectively, and the stress levels for the fatigue tests were taken at the interval of 2kg/mm² above fatigue limits as mentioned above.

Besides, the numerical values written near the plots mean the number of the superposed experimental results and the plots with arrow the run-out specimens.

From Fig. 6, it is observed that the fatigue strength of the joint specimen is much higher than that of the base material specimen as a whole and such a strength behavior can be recognized from the hardness distribution shown in Fig. 5, that is, the hardness value at the minimum sectional area of the joint specimen is considerably higher than that of the base material specimen. But the scatter in the fatigue life of the joint specimen seems to be considerably large in comparison with that of the base material specimen.

The characteristics of the fatigue failure life distribution for both types of the specimens would be evaluated below based on the configuration on the Weibull probability paper. Especially the difference in the three parameters of Weibull distribution function, *i.e.*, location parameter γ, scale parameter α and shape parmeter m will be compared and discussed.

In this study reported here, the statistical analysis is made by neglecting the contribution of the run-out specimens to the fatigue life distribution characteristics, for main purpose of this study is to clarify the difference in the fatigue failure life distribution between both types of the specimens used in this study.

To begin with, the statistical analysis with the single Weibull distribution function was tried based on the postulation that the fatigue failure life distribution at each stress level can be approximated by a single distribution function as follows,

$$P(N)=1-exp\left(-\left(\frac{N-\gamma}{\alpha}\right)^{m}\right) \quad \cdots\cdots\cdots\cdots(2),$$

where N is the number of stress cycles to failure and γ, α, m are the Weibull three parameters described above.

By using the least square method, the regression curves for the results at each stress level were obtained, and are shown in Fig. 7. The ordinate of Fig. 7 was calculated with Eq. (1),

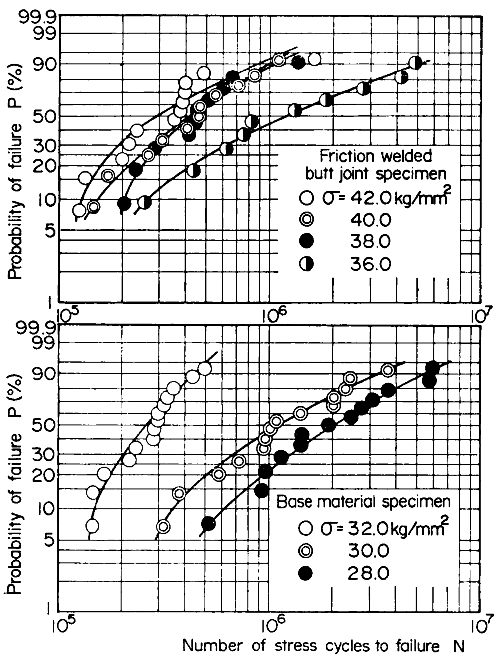

Fig. 7 Results plotted on Weibull probability paper and Weibull analysis with single distribution function.

but the value of n in denominator is taken as the total number of the broken specimen only. Furthermore, the variation trend of the three Weibull Parameters against the stress difference

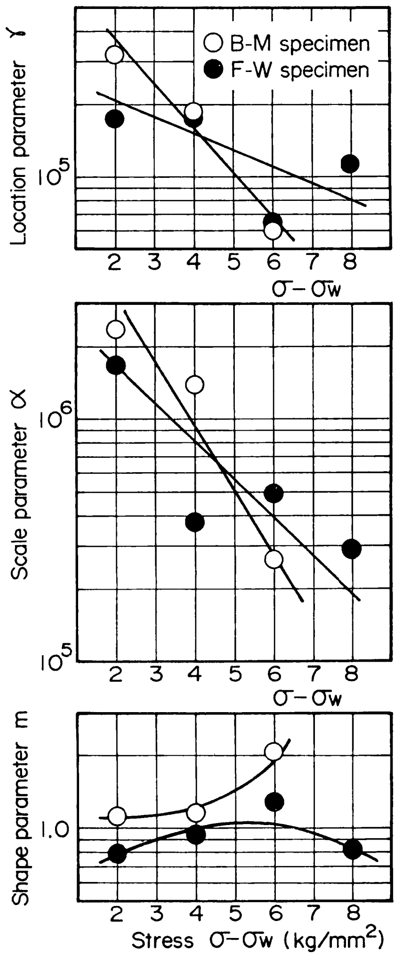

Fig. 8 Variation of Weibull parameters to stress $\sigma-\sigma_w$.

$\sigma-\sigma_w$ is represented in Fig. 8, where σ is the experimental stress level and σ_w the fatigue limit for both types of the specimens.

Fig. 8 shows that the variations of the two parameters γ, α tend to decrease gradually with the increase in the stress $\sigma-\sigma_w$ and there exists no remarkable defference between the behaviors of both types of the specimens. On the other hand, the distinct difference can be observed in the variation trend of the shape parameter m. That is, the joint specimen takes the values less than unity on an average in the range of this experiment despite that the base material specimen does the values of m>1 in the overstress range and shows a trend to take the value of m≦1 near or below the fatigue limit as is well known for ordinary metallic materials[17].

From the comparison of the Weibull parameters done above, it is concluded, depending on the value of m, that the fatigue failure lives of the joint specimen scatter in larger range than those of the base material specimen, when failure life distribution is approximated with a distribution function of Eq. (2).

Secondly, it seems to be more reasonable to adopt the mixed Weibull distribution function formulated in Eq. (3), when the configuration of the experimental results plotted on the Weibull probability paper is observed in more details. Examples of this procedure are illustrated in Fig. 9 (a) and (b).

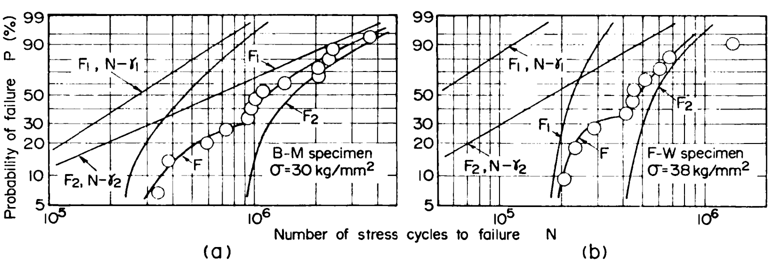

Fig. 9 Examples of Weibull analysis with mixed distribution function.

$$\left.\begin{aligned} P(N)=F(N)=p_1F_1(N)+p_2F_2(N) \\ p_1+p_2=1 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots(3).$$

In Eq. (3), $F_1(N)$ and $F_2(N)$ are the 1st and the 2nd distribution functions, and p_1, p_2 the probabilities of occurrence of $F_1(N)$ and $F_2(N)$ respectively. Hereafter, 1 and 2 mean the events of the 1st and of the 2nd distribution of fatigue life, where in a word the 1st distribution corresponds to the fatigue failure events which occur in relatively low stress cycles range and the 2nd distribution to those in relatively high stress cycles range.

Variation trends of the Weibull three parameters and of the probability of occurrence in mixed Weibull statistics analysis are represented in Figs.10 and 11 respectively.

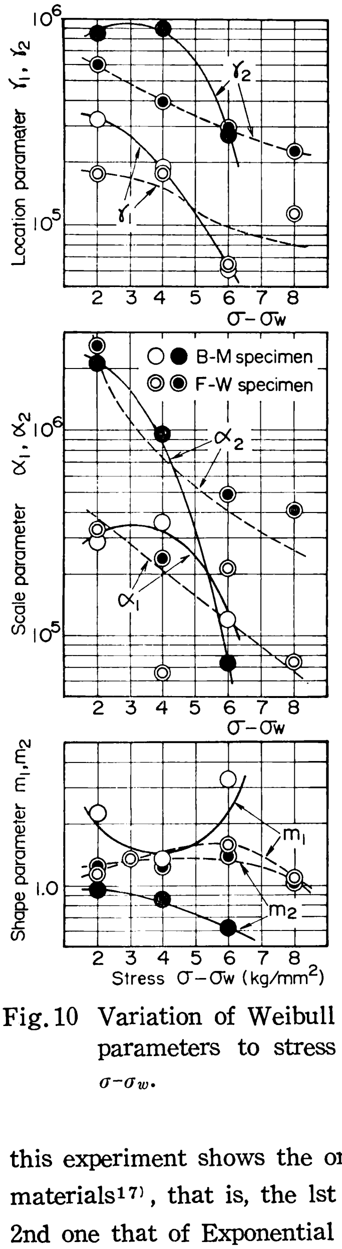

Fig. 10 Variation of Weibull parameters to stress $\sigma\text{-}\sigma_w$.

First, the distinct difference in these parameters between both types of the specimens is observed characteristically on the values of the shape parameters m_1 and m_2 in the same way as in the case of the single Weibull statistics analysis shown in Fig. 8. As isobserved in Fig. 10, the shape parameters for two distribution functions take the values of m_1, $m_2 > 1$ in the case of the joint specimen despite that the values of $m_1 > 1$, $m_2 < 1$ are taken for the base material specimen.

Next, it must be noticed that the attentionable discrepancy can be observed in Fig. 11. That is, the 1st distribution is predominant in higher stress range, but such a trend is reversed in lower stress range in the case of the base material specimen, on the other hand, as for the joint specimen both values of the occurrence probabilities, p_1 and p_2 take almost same value and the value of p_2 shows somewhat higher value than that of the 1st one in the whole range of the experiment.

The characteristic features of the probability density functions for several stress levels are evaluated for both types of the specimens based on the parameters shown in Figs. 10 and 11, and are represented in Fig. 12 schematically.

This figure reveals that the sequence of the probability density functions for both types of the specimens is a fundamentally different one depending on the difference in the value of the shape parameter of the 2nd distribution function. The difference in the fatigue failure life distribution behavior as seen above suggests that there exists some characteristic process in the initiation and the growth of the fatigue damage nuclei for the joint specimen, for the aspect of the fatigue failure life distribution of the base material specimen used in this experiment shows the ordinary or the well known feature for many sorts of the metallic materials[17], that is, the lst distribution function takes the form of Gamma type one and the 2nd one that of Exponential type. And, the detail examination on the fatigue process would be needed to clarify the fatigue failure life distribution characteristics of the joint specimen. But, such an approach from the metallurgical point of view having been out of the main interest of the study, any evaluation from this standpoint is not described in this paper.

Here, as for the base material specimen, the relationship between the fatigue failure lives and the hardness values was researched for the purpose of reference, but any correlation between them was not observed and the correlation factor was nearly equal to zero when this factor was evaluated for all of the broken specimens in disregard of the difference of the stress level experimented.

In the description done above, the nominal stress at the minimum sectional area of the

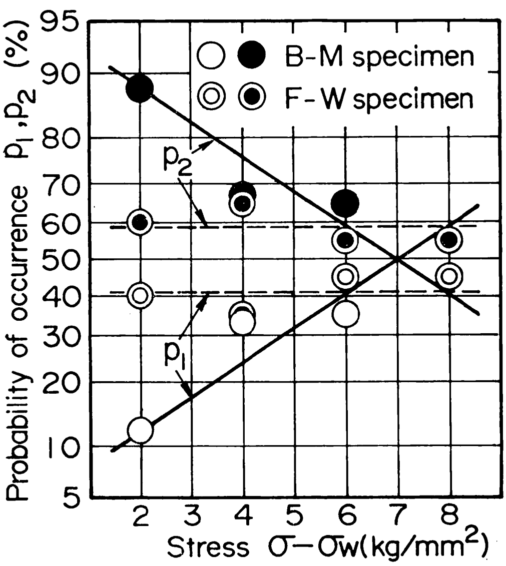

Fig. 11 Variation of probability of occurrence to stress σ-σ_w.

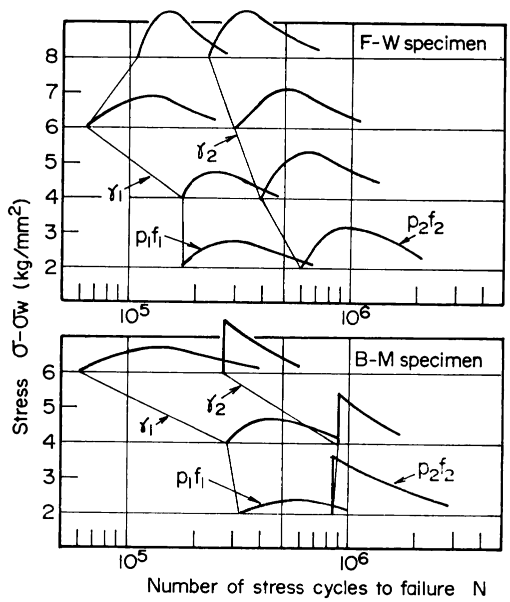

Fig. 12 Schematic representation of the S-N relation with the distribution of fatigue life.

specimen was used to discuss the fatigue life distribution characteristics of both types of specimens. However, it is worth noticing that tha fatigue failure does not occur at the root of the semi-circular notch in the case of the joint specimen, for the material at this location is hardened considerably as shown in Fig. 5. So the distance of the fracture location from the notch root was measured to obtain the real stress value, especially for the joint specimen broken under the nominal stress condition of σ=40kg/mm^2, and it was revealed that the distance from the notch root was about 1.98mm as a mean value, in other words, that the fatigue failure occurred in the HAZ referring to Fig. 5.

Additionally, the hardness value at the fractured location was also measured to estimate the fatigue life based on both values of the hardness thus obtained and the real stress calculated.

Here, the statistical relationship between the hardness Hv and the fatigue limit σ_w shown in Eq. (4)[19] was modified to the form shown in Eq. (5) by considering the results obtained in this experiment for the base material specimen, that is, the fatigue limit of about 26kg/mm^2 was given with the specimen of the hardness Hv=188.

$$\sigma_w = 0.17Hv - 0.97 \quad \cdots\cdots\cdots\cdots(4).$$
$$\sigma_w = 0.17Hv - 6.71 \quad \cdots\cdots\cdots\cdots(5).$$

By using the values of the real stress and the hardness at the fractured section, results for the joint specimen under the nominal stress condition of σ=40kg/mm^2 are rearranged and are plotted on an S-N diagram in Fig. 13, where the small solid circle means the plot for the cumulative probability for failure P=50% of the base material specimen shown in Fig. 6, the open circle that obtained by considering the real stress only and the double circle that obtained by considering both values of the real stress and the hardness. And a chain line means the estimated S-N relation for the hardness Hv=197 which is the mean hardness

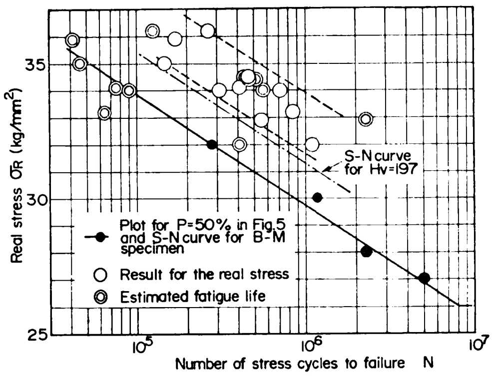

Fig. 13 Fatigue life of the friction welded butt joint specimen evaluated with the real stress and the hardness at the fractured section.

value at the fractured section of the joint specimen.

Besides, the fatigue lives plotted with the key ◎ were determined with the following manner; At first, *S-N* curve concerned with the corresponding hardness value was drawn to run parallel with the original *S-N* curve for the base material specimen after estimating the fatigue limit based on Eq. (5), then the intersecting point of this S-N curve with the real stress level was searched and this point was plotted with the key ◎. The *S-N* relation presented with a chain line was also determined with the same manner as above.

Fig. 13 shows that, as a whole, the fatigue lives for the real stress (○) are lying in longer life range than those obtained by considering the hardness value (◎), and furthermore exceed the *S-N* relation for the mean hardness value of $Hv=197$.

Whole strength feature of the friction-welded butt joint specimen shown above suggests that the *S-N* relation of this type of joint can not be predicted accurately from the ordinary concept that the fatigue strength is well correlated to the hardness of the specimen and that the metallugical characteristics induced during the friction welding process must be taken into account.

5. Conclusions

As a link of a chain to study the fatigue strength behavior of the friction welded butt joint specimen, a series of the experiments were carried out by using the joint specimen composed of a similar material, *i. e.*, 0.46%C carbon steel. And the results obtained were analised from the view point of Weibull statistics to clarify the fatigue failure life distribution characteristics of this type of joint specimen in comparison with those of the base material specimen.

Major conclusions obtained are as follows.

(1) At first, Weibull analysis done with single distribution function showed that the distinct difference in three Weibull parameters can be observed on the shape parameter m. That is, the parameter m takes the value less than unity in the case of the joint specimen as a mean value despite that the values more than unity are obtained for the base material specimen. Saying in other words, the scatterring of fatigue failure lives is larger for the joint specimen than for the base material specimen.

(2) Secondly, results of the mixed Weibull analysis demonstrated that the statistical difference in the fatigue failure life distribution between two sorts of the specimens used was also observed on the shape parameter and on the probability of occurrence for each distribution function.

As for the base material specimen, the 1st distribution function takes the value of $m_1>1$ and the 2nd one $m_2<1$. On the other hand, the parameters m_1 and m_2 take the values more than unity in the case of the joint specimen.

Value of the probability of occurrence takes almost constant value for each distribution function in the case of the joint specimen inspite of the variation of the stress level tested contrary to the well known general trend for carbon steels and other metallic materials, *i. e.*, the probability of occurence p_2 for the 2nd distribution function becomes predominant gradually with the decrease in the stress tested as is observed on the results for the base material specimen in this study.

From the description done above, it is seen easily that the configuration of both probability density function, p_1 f_1 and p_2f_2 shows somewhat different feature between the joint and the basematerial specimens.

(3) Through a rough evaluation of the fatigue failure lives of the joint specimens done based on the real stress and the hardness at the fractured location, it was revealed that the fatigue strength of this type of the joint specimen could not be estimated from the view point of the hardness only and that carefull research on the metallugical characteristics near the friction-welded interface would be needed to establish the successfull understanding on the fatigue strength behavior of this type of joint.

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APPENDIX*

Main program for single Weibull statistics analysis

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C       S45C STEEL ROTATING BENDING
C       FATIGUE TEST
        DIMENSION AN(45)
        DIMENSION F1(45),XSB1(45),Y0F1(45),
      1 Y1F1(45),YF1(45)
        ACCEPT "N2=", N2
        NN=N2
        TYPE "AN NO ATAI"
        READ(11,150) (AN(I),I=1,NN)
    150 FORMAT(E10.3)
        CALL PART1(NN, AN, F1, XSB1, YOF1,
      2 Y1F1,YF1,ESB1,VSB1,ZA2WA)
        XMIN=ZA2WA
        DGAM=2.0*10**3
        DO 20 M2=1,300
        DO 15 I=1,NN
     15 AN(I)=AN(I)-DGAM
        CALL PART2(NN, AN, F1, XSB1, Y0F1,
      3 Y1F1,YF1,ESB1,VSB1,ZN2WW)
        IF(XMIN.LT.ZN2WW) GO TO 30
        XMIN=ZN2WW
     20 CONTINUE
     30 DO 35 I=1,NN
     35 AN(I)=AN(I)+DGAM
        CALL PART1(NN, AN, F1, XSB1, Y0F1,
      4 Y1F1,YF1,ESB1,VSB1,ZA2WA)
        STOP
        END
```

Sub-program (1)

```
        SUBROUTINE PART1(N1,SB1,F1,XSB1,
      1 Y0F1,Y1F1,YF1,ESB1,VSB1,ZA2WA)
        DIMENSION SB1(45), F1(45), XSB1(45),
      2 Y0F1(45),Y1F1(45),YF(45)
        DIMENSION SB2(45), SG1(45), ZANSA
      3 (45), ZNSA2(45)
        DIMENSION FP(45),FC(45),FCP(45)
        DIMENSION CKSEN(45),CKSN1(45),
      4 CKSN2(45)
        DO 5 I=1,N1
        F1(I)=I/(N1+1.0)
        FP(I)=F1(I)*100.0
      5 CONTINUE
        L=N1-1
        DO 10 I=1,L
        K=I+1
        DO 10 J=K,N1
        IF(SB1(I).LE.SB1(J)) GO TO 10
        S=SB1(I)
        SB1(I)=SB1(J)
        SB1(J)=S
     10 CONTINUE
        WRITE(10,300) (SB1(I),I=1,N1)
    300 FORMAT(1H ,6H NF(I),E12.4)
        DO 15 I=1,N1
        XSB1(I)=ALOG(SB1(I))
     15 CONTINUE
        WRITE(10,350) (FP(I),I=1,N1)
    350 FORMAT(1H ,6H F(I), F12.2)
        DO 20 I=1,N1
        Y0F1(I)=1.0/(1.0-F1(I))
        Y1F1(I)=ALOG(Y0F1(I))
        YF1(I)=ALOG(Y1F1(I))
     20 CONTINUE
        CALL LSO(XSB1,YF1,N1,A,B)
        ALFA=1.0/A
        BETA=EXP(B)
        WRITE(1.0,400) ALFA,BETA
    400 FORMAT(1H ,5HALFA=,F9.2,5X,
      5 5HBETA=,E10.3)
        B2=ALOG(BETA)
        ZA2WA=0.0
        DO 25 I=1,N1
        CKSEN(I)=ALFA*(XSB1(I)-B2)
        CKSN1(I)=EXP(CKSEN(I))
        CKSN2(I)=EXP(CKSN1(I))
        FC(I)=1.0-1.0/CKSN2(I)
        FCP(I)=FC(I)*100.0
        ZANSA(I)=YF1(I)-CKSEN(I)
```

*With the courtesy of Mr.K.Okada, one of my good adviser for the study on the statistical treatment of fatigue problem. Fundamental program pattern listed here was offered from him.

```
      ZNSA2(I)=ZANSA(I)**2
      ZA2WA=ZNSA2(I)+ZA2WA
      WRITE(10,450) I,SB1(I), XSB1(I), YF1
     6 (I), CKSEN(I), FCP(I), ZASA(I),
     7 ZNSA2(I)
  450 FORMAT(1H ,I4,7E15.3)
   25 CONTINUE
      WRITE(10,451) ZA2WA
  451 FORMAT(1H ,73X,6HZA2WA=,E15.3)
      WRITE(10,405)
  405 FORMAT(1H0)
      RETURN
      END
```

Sub-program (2)

```
      SUBROUTINE PART2(N1,SB1,F1,XSB1,
     1 Y0F1,YF1,VSB1,VSB1,ZN2WW)
      DIMENSION SB1(45), F1(45), XSB1(45),
     2 Y0F1(45),Y1F1(45),YF1(45)
      DIMENSION SB2(45), SG1(45), ZANSA
     3 (45), ZASA2(45)
      DIMENSION FP(45),FC(45),FCP(45)
      DIMENSION CKSEN(45),CKSN1(45),
     4 CKSN2(45)
      DO 5 I=1,N1
      F1(I)=I/(N1+1.0)
      FP(I)=F1(I)*100.0
    5 CONTINUE
      L=N1-1
      DO 10 I=1,L
      K=I+1
      DO 10 J=K,N1
      IF(SB1(I).LE.SB1(J)) GO TO 10
      S=SB1(I)
      SB1(I)=SB1(J)
      SB1(J)=S
   10 CONTINUE
      DO 15 I=1,N1
      XSB1(I)=ALOG(SB1(I))
   15 CONTINUE
      DO 20 I=1,N1
      Y0F1(I)=1.0/(1.0-F1(I))
      Y1F1(I)=ALOG(Y0F1(I))
      YF1(I)=ALOG(Y1F1(I))
   20 CONTINUE
      CALL LSO(XSB1,YF1,N1,A,B)
      ALFA=1.0/A
      BETA=EXP(B)
      B2=ALOG(BETA)
      ZN2WW=0.0
      DO 25 I=1,N1
      CKSEN(I)=ALFA*(XSB1(I)-B2)
      CKSN1(I)=EXP(CKSEN(I))
      CKSN2(I)=EXP(CKSN1(I))
      FC(I)=1.0-1.0/CKSN2(I)
      FCP(I)=FC(I)*100.0
      ZANSA(I)=YF1(I)-CKSEN(I)
      ZNSA2(I)=ZANSA(I)**2
      ZN2WW=ZNSA2(I)+ZN2WW
   25 CONTINUE
      RETURN
      END
```

Sub-program (3)

```
      SUBROUTINE LSO(Y,X,N,A,B)
      DIMENSION Y(45),X(45)
      SUMX=0.0
      SUMY=0.0
      DO 10 I=1,N
      SUMX=SUMX+X(I)
      SUMY=SUMY+Y(I)
   10 CONTINUE
      AN=N
      XM=SUMX/AN
      YM=SUMY/AN
      SUMXX=0.0
      SUMXY=0.0
      DO 20 I=1,N
      SUMXX=SUMXX+(X(I)-XM)**2
      SUMXY=SUMXY+(X(I)-XM)*(Y(I)-
     1 YM)
   20 CONTINUE
      A=SUMXY/SUMXX
      B=YM-A*XM
      RETURN
      END
```

Program for mixed Weibull statistics analysis

```
C     F(N) F1(N) F2(N) NO KETTEI
      ACCEPT "SG=", SG
      ACCEPT "E1=", E1
      ACCEPT "A1=", A1
```

```
      ACCEPT "G1=", G1
      ACCEPT "E2=", E2
      ACCEPT "A2=", A2
      ACCEPT "G2=", G2
      ACCEPT "P1=", P1
      P2=1.0-P1
      WRITE (10,100) SG E1,A1,G1,E2,A2,G2,
    1 P1,P2
  100 FORMAT(1H ,3HSG=,F5.1,2X,3HE1=
    2 ,F5.2,2X,3HA1=,E10.3,2X,3HG1=,E10.3,
    3 2X,3HE2=,F5.2,2X,3HA2=,E10.3,2X,3HG
    4 2=,E10.3,2X,3HP1=,F5.2)
    5 2X,3HP2=,F5.2)
      DN=2.0*10**4
      AN=G1
    1 AN=AN+DN
      IF(AN.GT. 0.8E 07) GO TO 30
      B1=(AN-G1)/A1
      C1=-(B1**E1)
      F1=1.0-EXP(C1)
      IF(AN.LE.G2) GO TO 20
      B2=(AN-G2)/A2
      C2=-(B2**E2)
      F2=1.0-EXP(C2)
      F=P1*F1+P2*F2
      WRITE(10,200) AN,F1,F2,F
  200 FORMAT(1H ,4E15.3)
      GO TO 10
   20 F=P1*F1
      WRITE(10,300) AN,F1,F
  300 FORMAT(1H ,3E15.3)
   10 GO TO 1
   30 STOP
      END
```